Quantum Draft Final Submission

This paper will discuss the key differences between the classical and quantum interpretations of physics.

Classical physics describes the world in two very distinct ways. Objects behave as particles or objects behave as waves. The distinction as to which, is determined by certain characteristics displayed by the object. For example, wave-like properties include wavelength, frequency, and interference patterns. Particles, on the other hand, are localized, and can be defined by position and momentum. The classical interpretation of the world states that a body can act as either a particle or a wave. Never both.

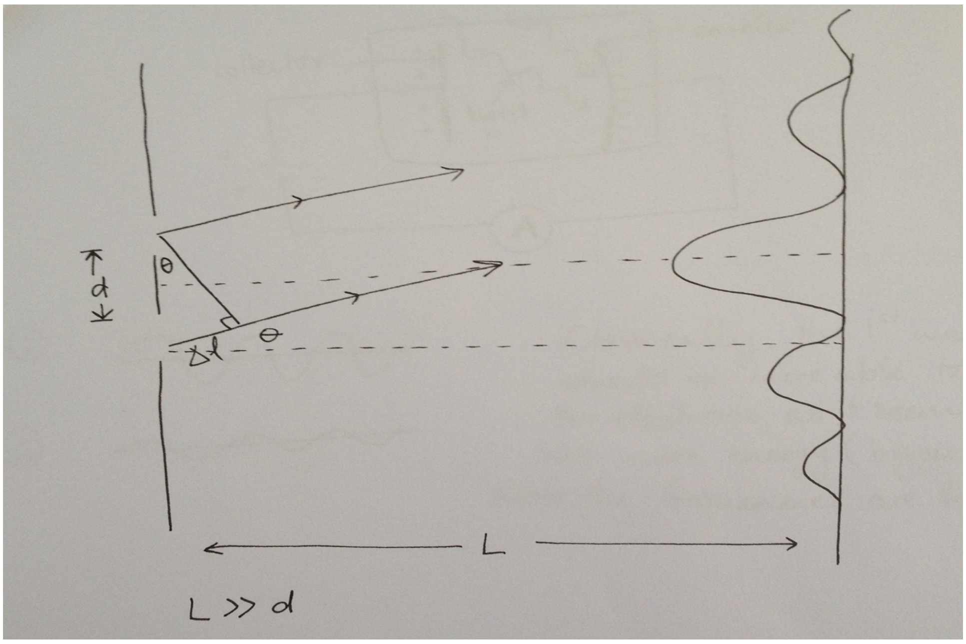
Another aspect of classical physics is that properties of an object can have continuous values. In other words, characteristics of the object such as momentum and energy, can take any value. These values can be known to arbitrary accuracy. That is, it makes sense to talk about the position and momentum of an object with absolute certainty.

The discovery of Quantum physics has demonstrated that this Classical view of the world is not quite complete. As further investigations take place, it becomes more and more apparent that the above assertions of physics can no longer explain certain experimental results.

This paper will begin with the discussion of wave-particle duality.

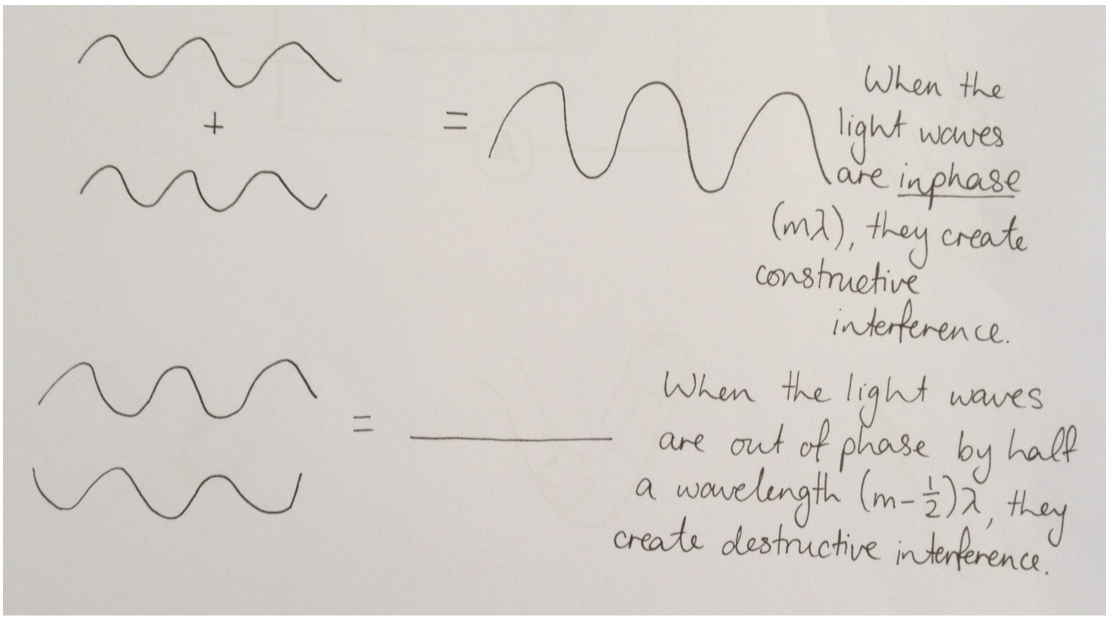
Wave-Particle Duality

Young's Double-slit experiment demonstrates the wave-like properties of light. The experiment consists of a beam of light shone through two neighbouring slits a small distance d apart, and hitting a screen a comparatively large distance L away:



Experimental evidence shows that the two light beams produced an interference pattern, with maxima at Δl=mλ and minima at Δl=(m-1/2)λ. This is shown in the diagram (where Δl is the difference between the light path lengths). These results display similar characteristics of a previous experiment that demonstrated how water waves interact with each other (Huyguen's Principle), producing an interference pattern with the maxima and minima in the same areas.

This concept can be explained by a simple diagram:



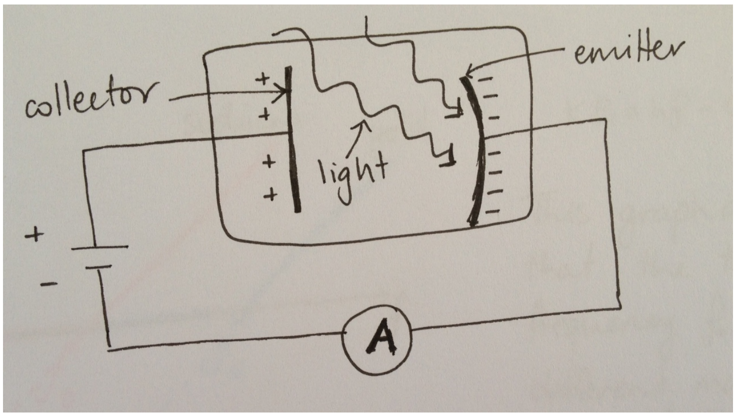
From these observations it was concluded that light was a wave.

But we know better than to assume the story ends there…

The Photoelectric Effect

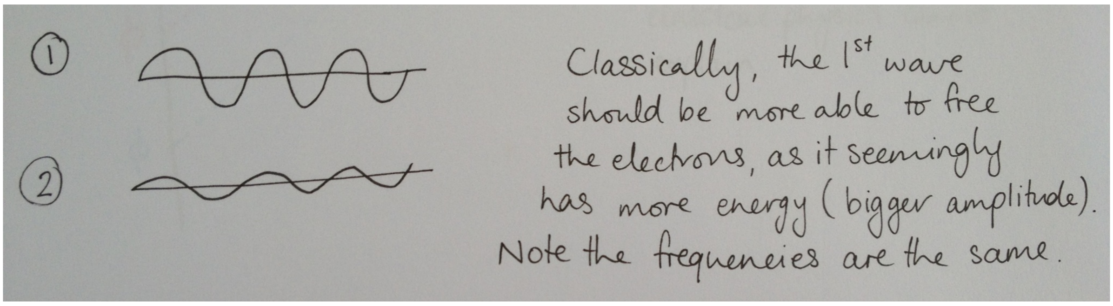
Later on, Einstein performed an experiment which disputed the assertion that light is a wave.

The experimental set-up consisted of two oppositely charged metal plates, placed in a vacuum tube. We will call the negatively charged plate the emitter and the positively charged plate the collector. The positive plate was connected to the positive terminal of the battery, which was connected to an ammeter. This is shown in the schematic. Light was shone onto the emitter, and if the light was of sufficient energy, electrons would be freed from the emitter plate and attracted over to the collector, producing a current.



Before we reveal the experimental results, let's first discuss what classical physics predicts should happen.
Regardless of the colour (frequency) of the light, if the intensity is high enough the electrons should be given enough energy to be freed. If we imagine light as a wave, as it hits an electron there should be a gradual (very fast but still not instantaneous) build-up of energy, allowing the electron to be emitted. This corresponds to electrons

being emitted if the amplitude of the wave is large enough. Let's take two blue waves for example:

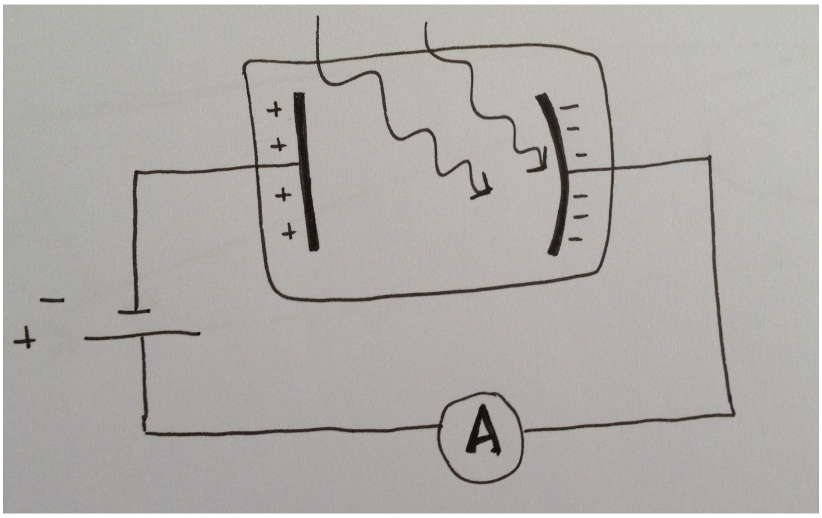


Secondly, classical physics states that the kinetic energy of the released electron should be greater if the amplitude of the wave is bigger. That is, if the energy transferred to the electron is greater, it should travel toward the collector plate at a higher velocity. So as A goes to ∞, KE goes to ∞.

Turns out classical predictions were wrong.

What actually happened:

To make the experimental results clearer, we will now flip the battery terminal, so that the negative terminal is connected to the collector as shown in the diagram (and as was done in the experiment).

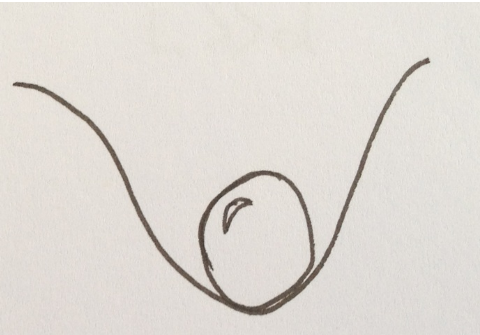


Note that the electrons in the emitter plate are no longer attracted to the collector, so in order for them to pass the gap they must be given a certain threshold energy (E_0).

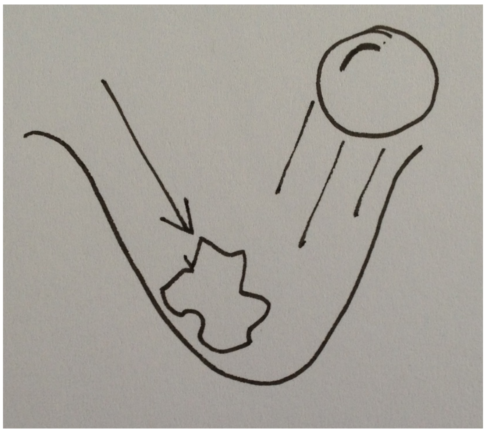
The experimental data showed that for an electron to be emitted, the light shone onto the emitter had to be of a high enough frequency (you can think of this as corresponding to colour). Anything below the threshold frequency (f_0) did not produce a current, regardless of intensity.

The only way this observation makes sense is if energy is delivered in discreet quanta. Einstein called these packets of light energy photons. The reason for this can be explained through a simple analogy:

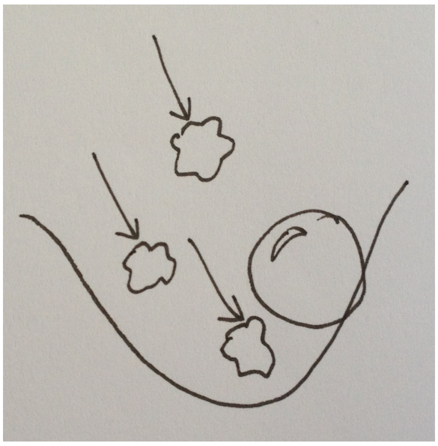
Imagine there is a ball in a well as shown.



If a rock is thrown at it with enough kinetic energy, it will bounce out.



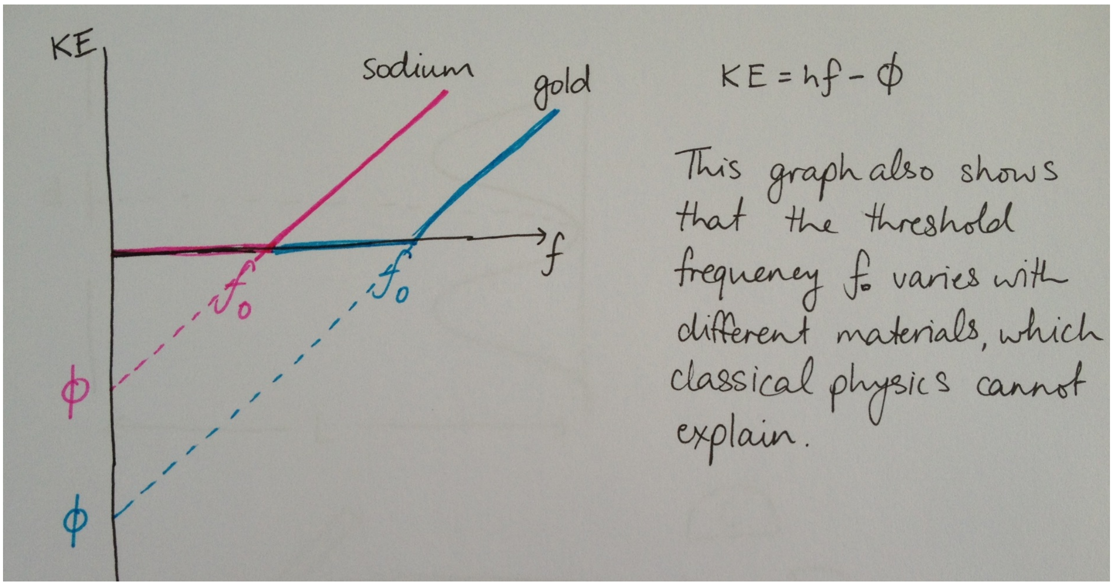
However if a rock is thrown with a kinetic energy lower than what is needed to knock the ball out, it will simply keep rolling up the sides and never be free. Even if we throw more rocks at it (increase intensity), if they aren't of sufficient energy the ball will never bounce out.



Likewise with the electrons on the metal surface, if the frequency of the light is below the threshold frequency, the electrons will not receive enough kinetic energy to be knocked off the emitter.

It is also an important observation that although an increase in intensity did not correspond to an increase in kinetic energy, when f exceeded f_0, a greater intensity corresponded to a larger current. This is because each photon interacts with only one electron, so more photons simply means more photon-electron interactions.

The kinetic energy of the electron as it travelled toward the collector is given by KE=hf-φ, where h is planks constant, f is the frequency of the light, and φ is the work function (φ=hf_0, or in other words the minimum amount of energy required to knock the electrons off the metal surface). A graph is shown below:

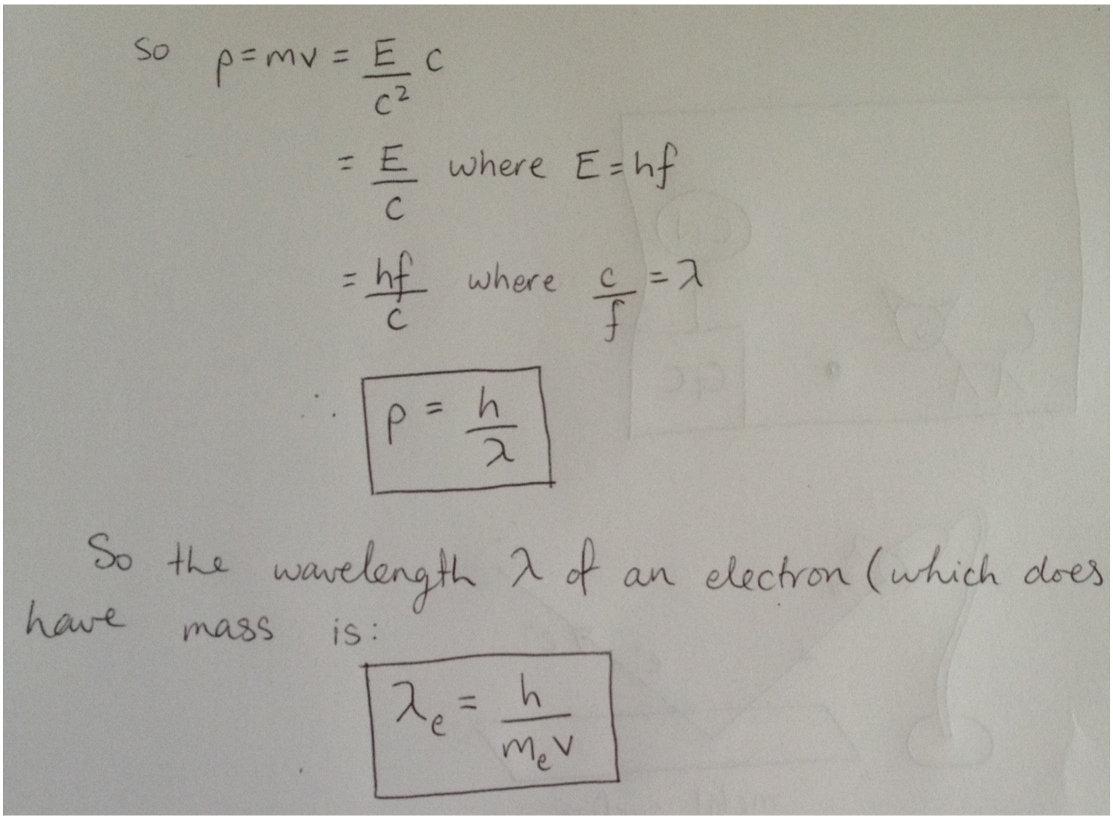


So here we have the first signs of energy quantization, light acting as a particle and the Einstein relation E=hf.

De Broglie

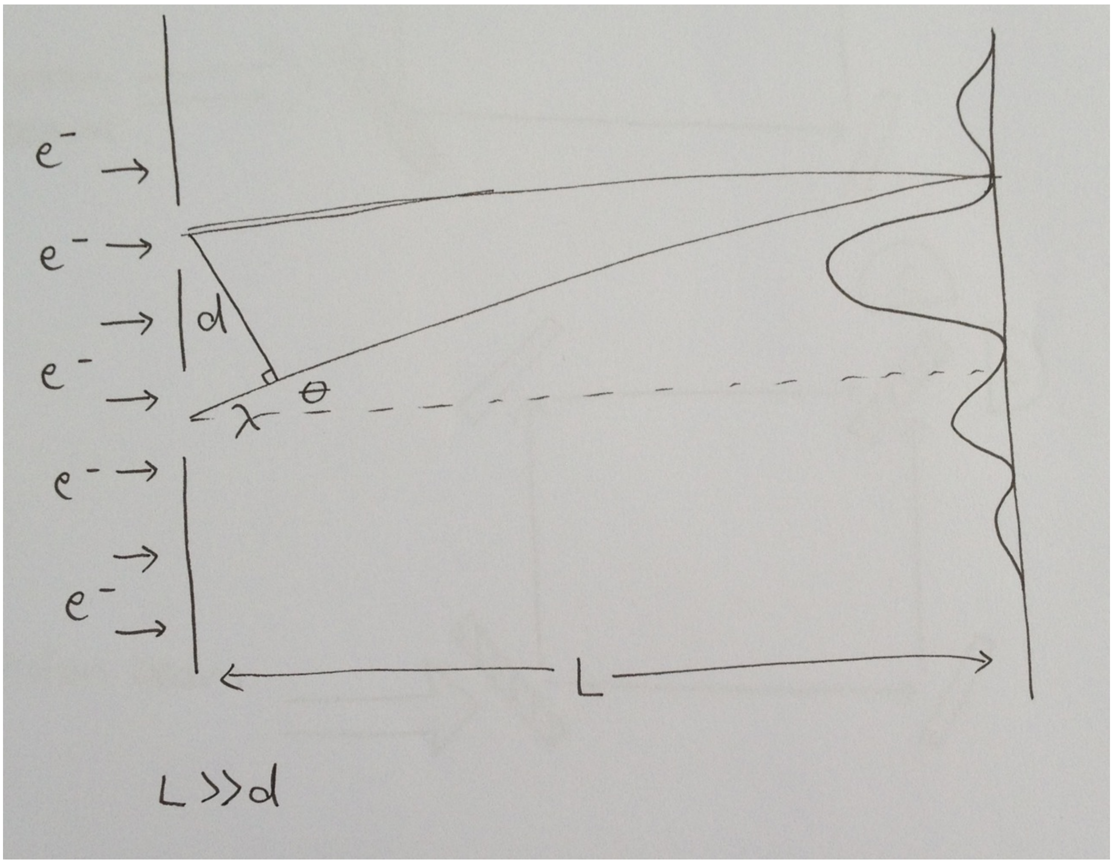
After discovering that light (generally thought of as a wave) displayed particle like properties, De Broglie made the daring assumption that electrons (classically thought of as particles) could perhaps act as waves.

De Broglie came up with an expression for the momentum (classically a property of a particle) of a wave. As photons do not have any mass, he let $m_{photon}=E/c^2$ from Einstein's $E=mc^2$.



Experimental evidence of this relation can be seen from the Hitachi Double-slit experiment.

The Hitachi double-slit experiment consists of electrons being shot at a screen through two neighbouring slits, separated by a small distance d. Electrons are classically thought of as particles, as they have mass, momentum and are localized (close to localizable anyway). However, instead of producing two distinct central maxima as classical physics would predict, a diffraction pattern was produced on the screen.



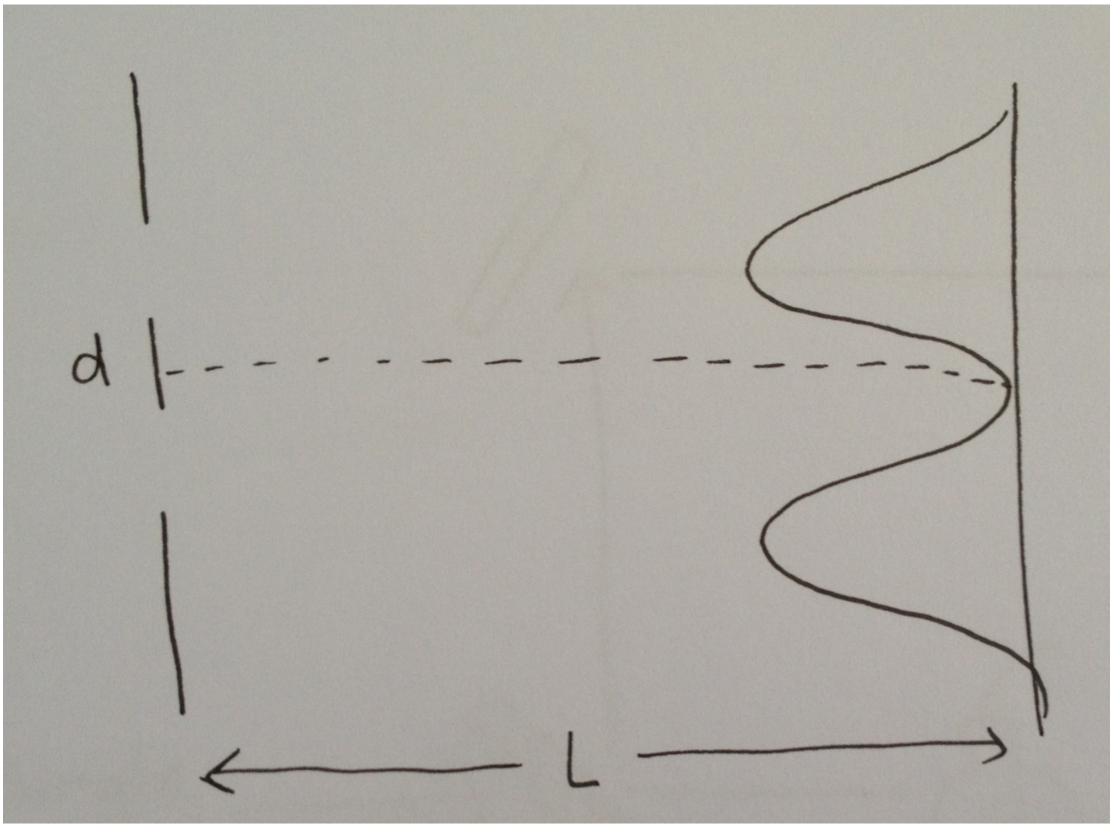
The wavelength of the electrons was measured using $\lambda=/mv$, and it corresponded with the calculations of the central maxima and minima using $\lambda=d\sin\theta$.

So confirming De Broglie's assertion, we now have particle's displaying wave-like properties.

The most interesting observations about the Hitachi double-slit experiment are as follows:

1. The experiment was repeated, except this time the electrons were shot out one at a time. An interference pattern was still produced. Now, we have concluded thus far that wave and particle properties can overlap, but these conclusions in no way explain how the electrons can form an interference pattern when they are travelling one at a time. After all, what are they interacting with?

2. To see what was going on, a detector was placed at the entrance of the slits, to monitor the path of the electron. When this happened, the electrons began behaving as particles ie. They formed two distinct maxima on the screen, as would be expected in classical physics.



These experimental results lead us to our next section and what is often referred to as the “heart” of quantum physics.

The Observer Effect

The key idea behind the observer effect is that before any system is measured, it exists in a superposition of states. Relating this concept back to the Hitachi double-slit experiment, we notice that when the electrons are not being monitored, the form an interference pattern. But what is the single electron interfering with? The answer is the electron is interfering with itself. It is passing through both slits, existing in a superposition of states. But, when we observe which path the electron takes, it is forced to "choose" which slit it goes through, and no longer interferes with itself. The mere act of observing has changed the outcome of the system, and caused the wave function to collapse. What do I mean by wave function? This will be briefly discussed in the next section, as it will help us to understand some concepts later on.

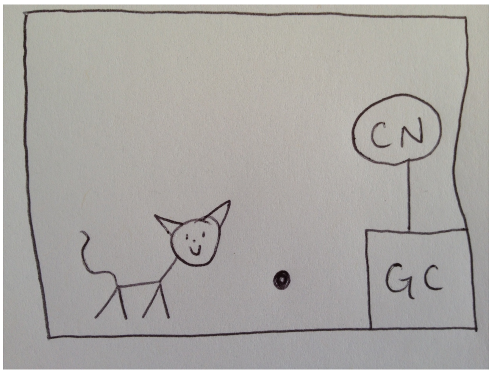
Schrodinger's Cat

I am considering omitting this section. Please let me know your thoughts.

Schrodinger's cat is a famous thought experiment, and is being included here because in order to go on we need to introduce the concept of a wave function.

The focus of this experiment is the idea of superposition- an object existing in conflicting states simultaneously.

The Experiment- A cat is placed in a box (as shown below) along with a Geiger counter. There is a tiny bit of radioactive substance, which has an equal probability of decaying and not decaying in the next hour. If the substance does decay, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanide, killing the cat.



The quantum physical description of the scenario states that, until you measure the outcome of the experiment (open the box and look), the atom exists in a superposition of states (decayed and not decayed) meaning that the cat also exists in a superposition of states (dead and alive). This of course conflicts with the classical interpretation of the scenario, as we are now dealing with probabilities, as we have only a probabilistic view of the state of the atom/cat before opening the box. This probability is referred to as the wave function.

The wave function of the atom says it is both decayed and not decayed at the same time, because we really cannot predict the outcome either way. Only when we measure the outcome (look in the box) do we know for certain the state of the atom, causing the wave function to collapse (as we exit the world of probabilities).

This thought experiment was designed to compare the view of the quantum world vs classical world. Classical physics would say the cat is either dead or alive, independent of measurement. Quantum physics is saying that the cat exists in a superposition of being dead and alive, until we measure it. In other words, the outcome of the situation depends on the measurement.

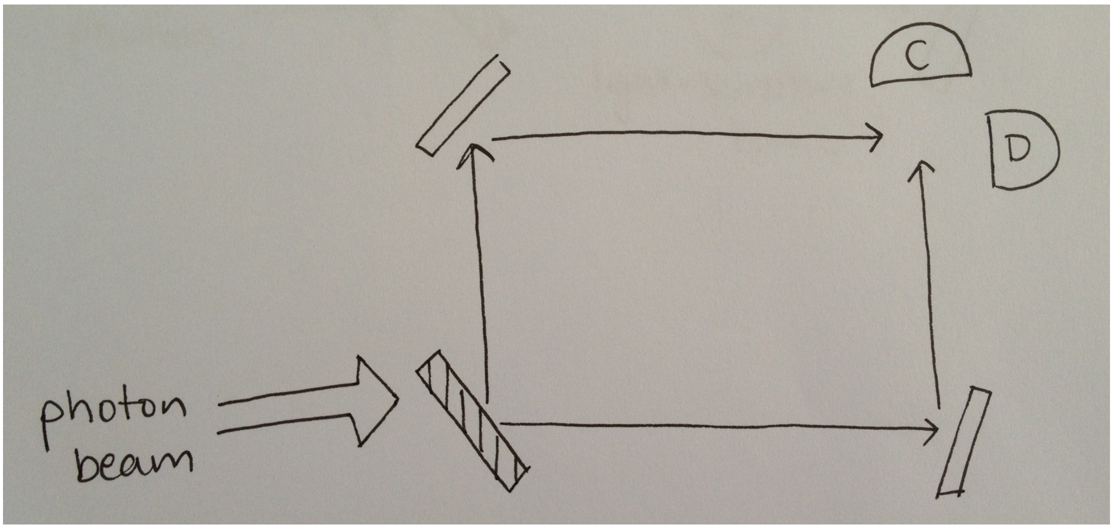
*Note: From the cat's perspective, the wave function of the atom never existed. She is always certain whether or not the atom has decayed or not (unless she is dead, in which case she lacks the capacity to be certain of anything).

At first, this idea of a wave function and superposition of states may seem like merely an interesting way of thinking or more philosophy based than physical.

However it turns out that its implications are apparent in the physical world. A good example of this is the Mach-Zender Inferometer.

The Mach-Zender Inferometer

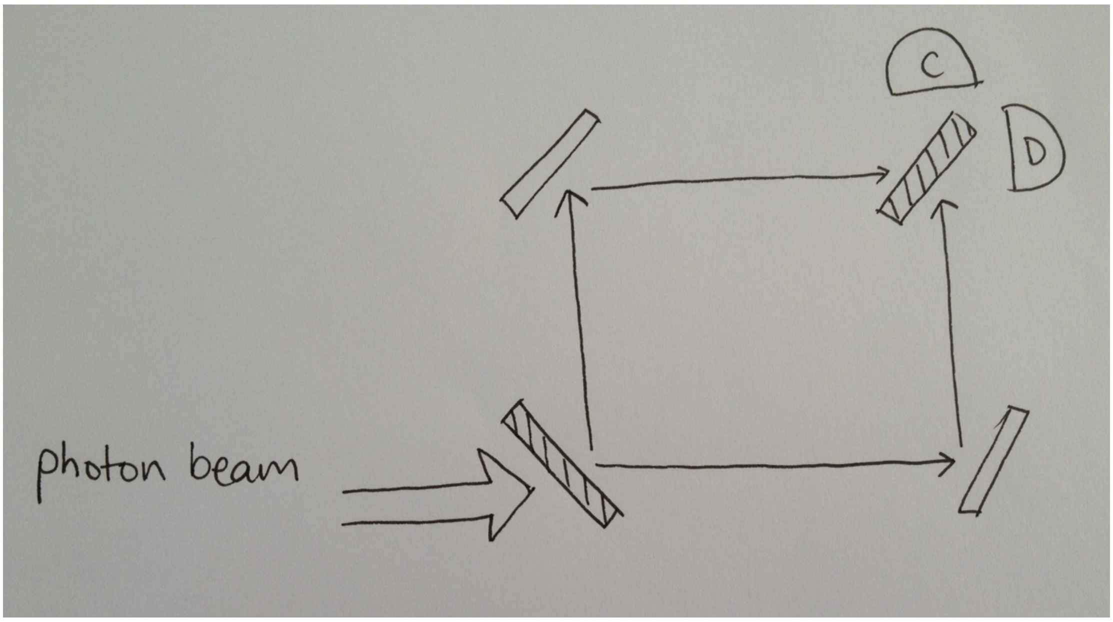
The Mach Zender Inferometer consists of a photon beam being shone into a beam splitter (lower left corner).



The beam splitter is set so there is a 50% chance a photon will take the lower path and a 50% chance it will take the upper path. Two mirrors are placed so that they reflect the light onto two different detectors (see diagram).

As would be classically expected, ~50% of photons are detected on each of the detectors. This is because "which path" information can be extracted simply by monitoring which detector the photon hits. For example, if the photon has landed on detector C, it clearly came from the lower path.

Now, another beam splitter is added just before the detectors:



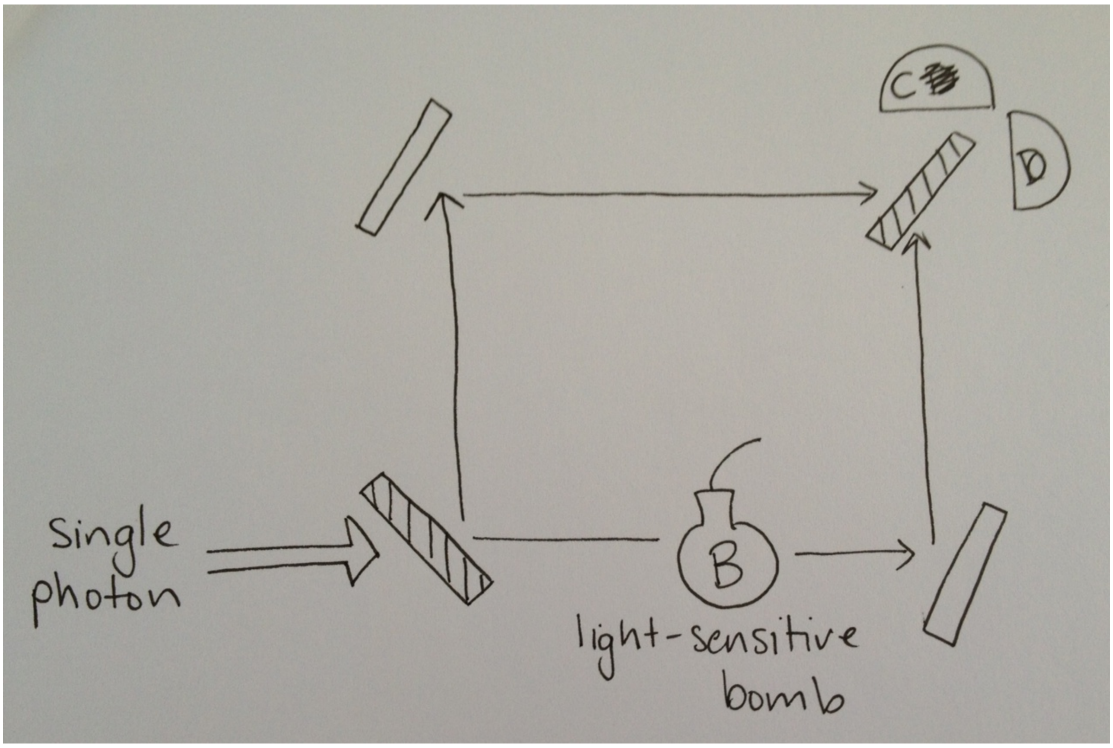
It is no longer clear which path the photon took, regardless of measurement. That is, if a photon is again detected on D, this tells us nothing about which path it took, due to the second beam splitter.

Following the predictions from the observer effect, this scenario produces a diffraction pattern on the detectors, as the photon chooses both paths and interferes with itself. Hence, the mere act of measurement forced the photon to choose a path, and changed the behavior of the system (forcing the wave function to collapse).

A solid application of this experiment is the Elitzur-Vaidman bomb tester.

<u>The Elitzur-Vaidman Bomb Tester</u>

This method was used to test whether certain light sensitive bombs were duds or real. The procedure was set up as follows:



A single photon is emitted from the photon beam and has a 50% chance of taking either the upper or lower path. The second beam splitter (top-right corner) is set so that, if an interference pattern is produced, constructive interference would be detected by detector C and destructive interference would be detected by D. This means the pattern would show up on C and D would remain blank. The following situations can occur.

If the bomb is a dud-

If the bomb is a dud, an interference pattern will form, as from the photons point of view it is equivalent to there being no bomb there at all. In other words, the photon can take either path without giving away any information about which it took.

If the bomb is real-

If the bomb is real, two things can happen:

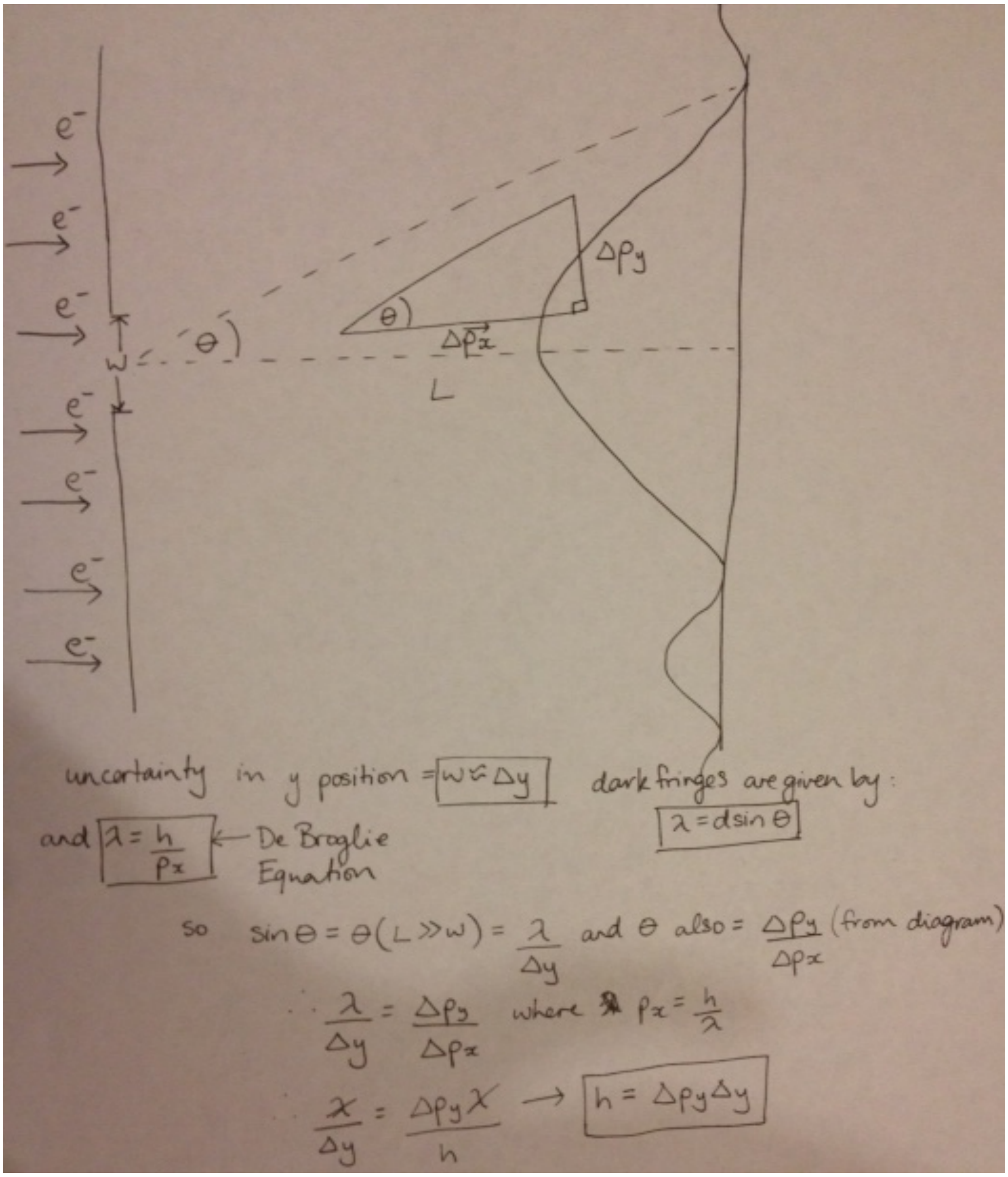
1. The photon takes the lower path, and the bomb detonates.
2. The photon takes the upper path, and lands on detector C or D. The photon is now forced to choose a path, as by the act of the bomb not detonating, it has given away

information about which path it took. It can no longer be in a superposition of states, as this would mean the bomb both does and does not detonate.

Remember that if the bomb is a dud, it is impossible that any photons land on detector D (destructive interference). Therefore, if a photon is detected on D, we can conclude it is indeed real. So 75% of the time we can infer whether a bomb is real without detonating it (this is better than nothing!).

The Heisenberg Uncertainty Principle

Closely related to the Observer Effect is the Heisenberg Uncertainty Principle. This principle states that the position and momentum of an object can never be known to 100% accuracy, and is demonstrated below with the single-slit experiment.



The experiment is much like the Hitachi double-slit experiment, except instead of there being two slits there is only one.

According to the classical definition of a particle, the electrons should all build up in a narrow space on the screen, as that is how particles behave classically. However we see an interference pattern, with probabilities of electrons landing in places on the screen far outside the width of the slit. We have already asserted this is characteristic of wave behavior.

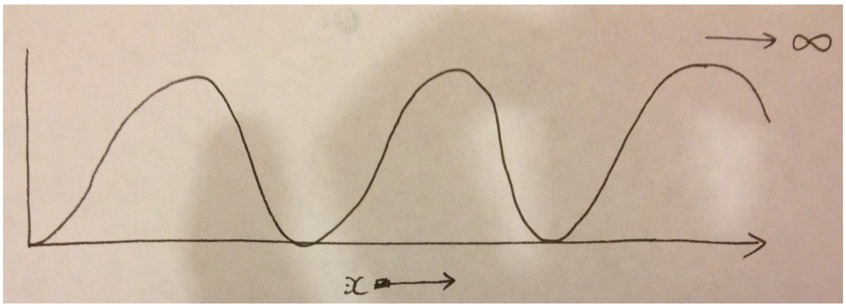
This demonstration has led us to another important principle related to Quantum physics. That is, the Heisenberg Uncertainty Principle, given by

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

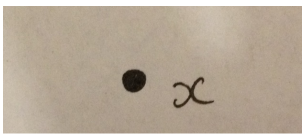
where Δx is the uncertainty in position, Δp in the uncertainty in momentum, and ħ is h/2π where h is Planck's constant. Although the above example doesn't prove these exact figures due to some small inaccuracies, the overall idea is the same.

In short, the Heisenberg Uncertainty Principle states the more we know about the position of an object, the less we know about the momentum (or velocity as they are directly related by mass) of that object and vice versa.
Extracting from the formula, we can see that Δx and Δp are inverses of each other, or "conjugate variables".

We will now examine the characteristics of waves and particles in more detail, and see how the Heisenberg Uncertainty Principle arises from these properties. Waves are very spread out:

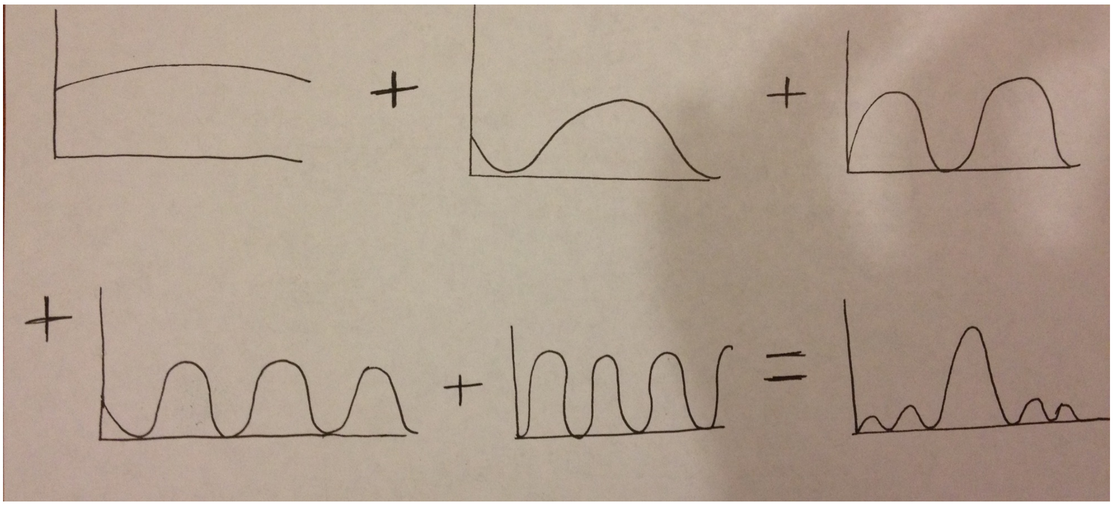


and particles are very localized:



If an electron were somewhere in the vicinity of the wave, this does not really help us to find the position of the electron as the wave is infinite. Thus Δx is infinite (or extremely close to infinity). However, remember that $p=h/\lambda$, so as there is only one wavelength present, we are certain about the momentum of the electron.

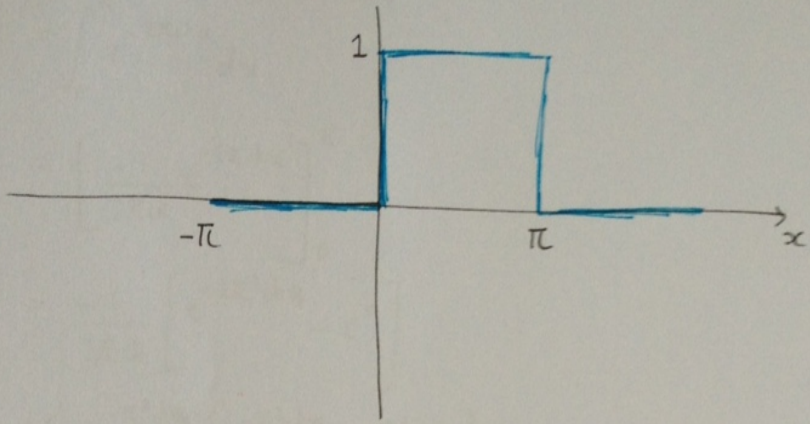
Now let's see if we can try and localize the wave. We can do this by adding multiple sine waves together:



The resultant wave is large within only a small region of space, so we have somewhat localized our wave, and now have a better idea of where the electron might be. However, we have now lost information about our momentum as we have used multiple wavelengths. Therefore, to obtain information about the position of the electron we must trade information about the momentum of the electron and vice versa.

This is just a simple example to demonstrate the underlying meaning behind the Heisenberg Uncertainty Principle. A more formal example is shown below.

Fourier Transform of a Square wave



We start with a square wave in position space. We make the amplitude 1 and the wavelength $\lambda = 2\pi$. The area under the square is representative of the electrons <u>position</u>. This means, although we do not know the exact location of the electron, we know that it lies between $x=0$ and $x=\pi$. So the probability of finding the electron $0<x<\pi$ is equal to 1, and $x<0$ and $x>\pi$, the probability of finding it is zero.

However, because this wave is made up of infinite wavelengths, from $p = \frac{h}{\lambda}$, we know nothing about the momentum of the particle.

Using the Fourier Transform, let's see what happens when we turn this function of position ($f(x)$) into a function of wave number ($f(k)$), in order to find out more information about the momentum of the particle. We use k as it is closely related to the wavelength ($k = \frac{2\pi}{\lambda}$) and momentum ($p = \hbar k$). It is also a conjugate variable of x. A similar comparison can be seen with time and frequency, where $f = \frac{1}{T}$.

I don't think this is the right explanation.

$$f(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx$$

$$= \int_{0}^{\pi} e^{-2\pi i k x} dx$$

$$= \left[\frac{-1}{2\pi i k} e^{-2\pi i k x} \right]_0^{\pi}$$

$$= \frac{-1}{2\pi i k} \left[e^{-2\pi^2 i k} - e^{0} \right]$$

$$= \frac{-e^{-\pi^2 i k}}{2\pi i k} \left[e^{-\pi^2 i k} - e^{\pi^2 i k} \right]$$

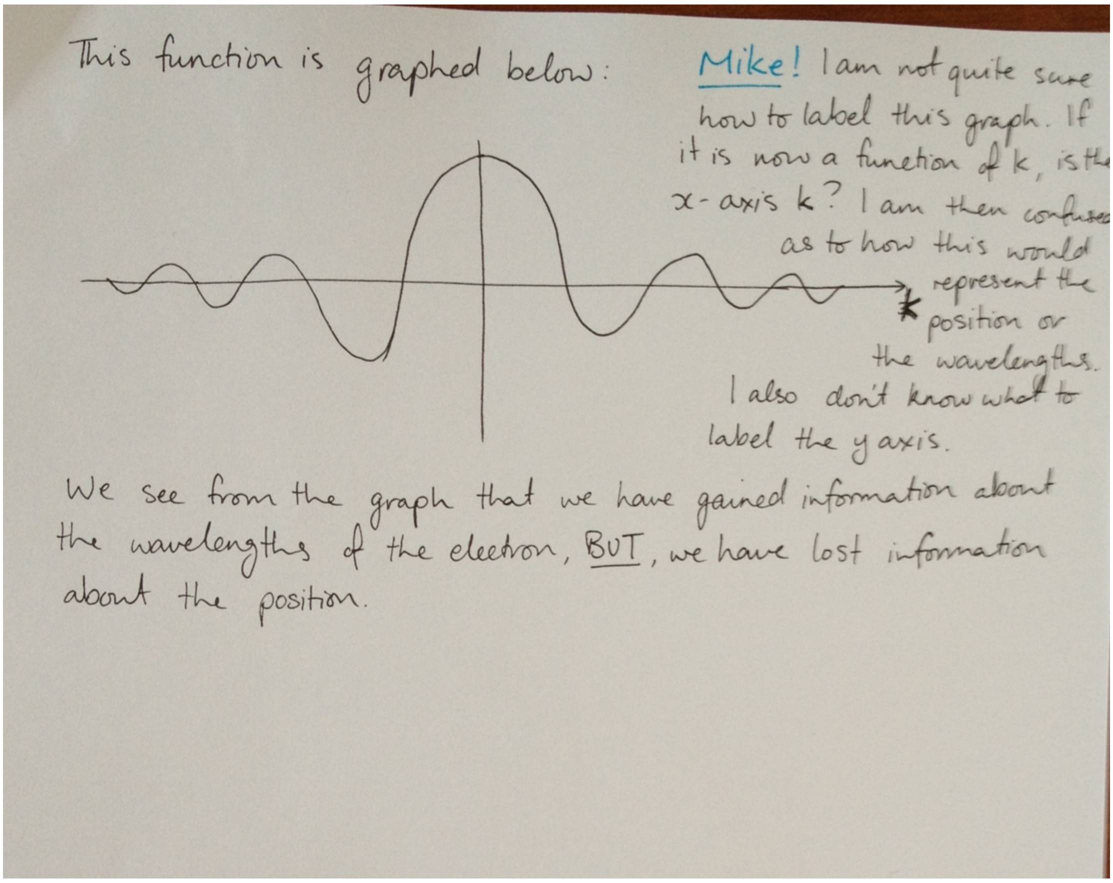
$$= \frac{e^{-\pi^2 i k}}{\pi k} \left[\frac{e^{\pi^2 i k} - e^{-\pi^2 i k}}{2i} \right] \Bigg\} \quad \frac{e^{x} - e^{-x}}{2i} = \sin x$$

$$= \frac{e^{-\pi^2 i k}}{\pi k} \sin \pi^2 k \quad \longrightarrow \quad \text{sinc}\, x = \frac{\sin \pi x}{\pi x}$$

$$= e^{-\pi^2 i k} \frac{\sin \pi^2 k}{\pi k}$$

$$= \pi e^{-\pi^2 i k} \text{sinc}\, \pi k$$

$$\therefore \boxed{f(k) = \pi e^{-\pi^2 i k} \text{sinc}\, \pi k}$$



So why is this Principle not apparent in everyday life (or the classical scale)?
Well:

$p=mv$ and $\Delta x \, \Delta p \geq \hbar/2$, which gives $\Delta x \geq \hbar/2m\Delta v$

So from this relation we conclude that for large masses (macroscopic objects) the uncertainty in position if practically negligible. However, on the quantum scale where masses become very small, the uncertainty in x becomes more significant.

Probability Densities

Although we cannot know with exact precision where the particle is, we can know the probability of it being in a certain area. Take for example the square wave

from the previous page. We see that when $x<0$ and $x>\pi$, there is zero probability the electron will be found as there is zero area under the curve. However, when $0<x<\pi$ the electron has an equal probability of being found for all values of x. Note that because the electron must be somewhere in the vicinity of $0<x<\pi$, the sum of all the probabilities of the x coordinates must be 1.

So how do we go about finding these probability densities? We will now introduce the Schrodinger equation.

The Schrodinger equation

In Quantum mechanics, the information about the system is contained in the solution to the Schrodinger equation, where the square of the function is interpreted as the probability density. It tells us virtually everything we need to know about the system, and is derived below (in one dimension):

$E = PE + KE \Rightarrow E = \frac{1}{2}mv^2 + V$ and $p = mv$

$$\therefore E = \frac{p^2}{2m} + V$$

we have established that $\Psi = e^{i(kx - \omega t)}$ (this is a more general form of the previous equation, including also travelling waves)

$$\therefore \frac{d\Psi}{dx} = ike^{i(kx-\omega t)} = ik\Psi$$

$$\frac{d^2\Psi}{dx^2} = i^2k^2e^{i(kx-\omega t)} = -k^2\Psi$$

but remember that $k = \frac{p}{\hbar}$

$$\therefore \frac{d^2\Psi}{dx^2} = \frac{-p^2}{\hbar}\Psi$$

$$\therefore -\hbar^2 \frac{d^2\Psi}{dx^2} = p^2\Psi$$ but $E = \frac{p^2}{2m} + V$

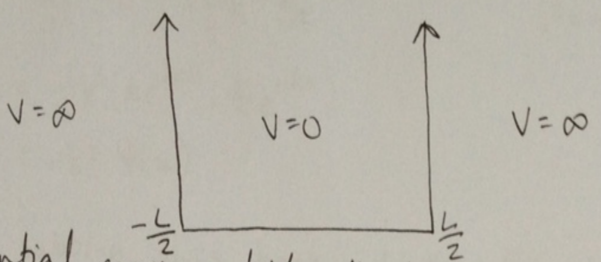
$$\therefore E\Psi = \frac{p^2\Psi}{2m} + V\Psi$$

$$\therefore \boxed{E\Psi = \frac{-\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V\Psi} \leftarrow \text{Time Independent Schrödinger Equation}$$

As a close to this paper, a mathematical example will be demonstrated to put into practice all the ideas previously discussed.

Particle in a Box

We have a particle in a box of infinitely high walls.



The potential energy outside the box is $V = \infty$, so the only way this can mathematically make sense is if the potential energy inside the box is $V = 0$.

So the ~~wave function~~ Schrödinger equation outside the box is

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + \infty\,\Psi(x) = E\Psi(x)$$

However, this expression must equal 0, as there is 0 probability the electron is outside the box.

So the electron is in the box, but where? Because this is a quantum particle, we can only find the probability densities of where the electron is likely or unlikely to be. Solving the Schrödinger equation for this problem will tell us all the information we need to know about the system:

$$\boxed{\frac{-\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} = E\Psi(x)}$$

Because we are now speaking in terms of probabilities, we will treat the electron as a <u>wave</u>. The most general wave formula is given by $Ae^{ikx} + Be^{-ikx}$. This would be a good guess, so let's try it out.

let $\Psi(x) = Ae^{ikx} + Be^{-ikx}$

$\therefore \frac{d\Psi}{dx} = (ik)Ae^{ikx} - ikBe^{-ikx}$

$\frac{d^2\Psi}{dx^2} = i^2k^2Ae^{ikx} + i^2k^2Be^{-ikx}$ $\quad i^2 = -1$

$\therefore = -k^2(Ae^{+ikx} + Be^{-ikx})$

$= -k^2\Psi(x)$

$\therefore$ subbing this back into the Schrödinger equation give

$$\frac{\hbar^2}{2m}k^2\Psi(x) = E\Psi(x)$$

so then the energy of the system is given by $E = \frac{\hbar^2k^2}{2m}$

From this relation, we ~~do not yet have~~ cannot yet conclude that the energy within the system is quantized. Solving for k will make this clear.

<u>Boundary Conditions</u>

We know that at $x = -\frac{L}{2}$ and $x = \frac{L}{2}$, $\Psi(x) = 0$, as there is 0 probability of finding the electron at the boundaries of the box. So subbing in $x = \frac{L}{2}$ gives

$\therefore \Psi(x) = Ae^{ikx} + Be^{-ikx} = 0$ when $x = \frac{L}{2}$ and $x = -\frac{L}{2}$

$\therefore \Psi(x) = Ae^{ik(\frac{L}{2})} + Be^{-ik(\frac{L}{2})} = 0$ using $e^{x} = \cos x + i\sin x$

$\Rightarrow A\cos\left(\frac{kL}{2}\right) + Ai\sin\left(\frac{kL}{2}\right) + B\cos\left(\frac{kL}{2}\right) - Bi\sin\left(\frac{kL}{2}\right) = 0$

$\Rightarrow (A+B)\cos\left(\frac{kL}{2}\right) + (A-B)i\sin\left(\frac{kL}{2}\right) = 0$ —— ①

Now subbing in $x = -\frac{L}{2}$

$$\Rightarrow A\cos\left(\frac{kL}{2}\right) - Ai\sin\left(\frac{kL}{2}\right) + B\cos\left(\frac{kL}{2}\right) + Bi\sin\left(\frac{kL}{2}\right) = 0$$

$$\Rightarrow (A+B)\cos\left(\frac{kL}{2}\right) - (A-B)i\sin\left(\frac{kL}{2}\right) = 0 \quad \text{——} \ (2)$$

Rewriting equations (1) and (2) as:

$$(1) \quad A\cos\left(\frac{kL}{2}\right) + B\sin\left(\frac{kL}{2}\right) = 0$$

$$(2) \quad A\cos\left(\frac{kL}{2}\right) - B\sin\left(\frac{kL}{2}\right) = 0$$

Mike! I don't understand how you arrived at these equations from the above!

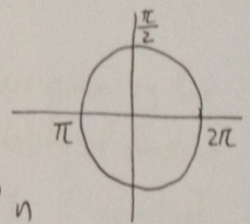
Now solving for k...

$$(1) + (2) = 2A\cos\left(\frac{kL}{2}\right) = 0$$

$$\therefore \cos\frac{kL}{2} = 0$$

$$\therefore \frac{kL}{2} = \frac{n\pi}{2}$$

$$\therefore k = \frac{n\pi}{L} \quad \text{for odd integers of } n$$



(1) − (2)

$$\Rightarrow 2B\sin\left(\frac{kL}{2}\right) = 0$$

$$\therefore \frac{kL}{2} = n\pi$$

$$\therefore k = \frac{2n\pi}{L}$$

OR $k = \frac{n\pi}{L}$ for even integers of n.

If we solve for the coefficients A and B, we can demonstrate graphically that only certain wavelengths (and therefore energies) can exist within the box, which we will do momentarily. However now would be a good point to return back to our initial energy expression

$$E = \frac{\hbar^2 k^2}{2m}$$

, subbing in k ~~...~~ this becomes

$$E = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$$

It is important to note that everysingle ~~..~~ term in this expression is either a constant, or integer value. From this we see that energy is quantized.

Now we will solve for coefficients A and B to see how this is physically possible.

Remember that the probability densities are given by the square of the wave function. Also, the probability of finding the particle ~~...~~ inside the box is equal to 1. If you look everywhere in the box, you physically have to find it, so we will set our probability $=1$. This is called normalizing.

We will split the wave function into 2 terms, starting with

$$\Psi(x) = A\cos\left(\frac{\pi n}{L}x\right)$$

$$P = |\Psi(x)|^2 = A^2\cos^2\left(\frac{\pi n}{L}x\right) = 1$$

$$\therefore \int A^2\cos^2\left(\frac{\pi n}{L}x\right)dx = 1$$

using $\cos^2 x = \frac{1+\cos 2x}{2}$

$$\therefore \frac{A^2}{2}\int 1 + \cos\left(\frac{2\pi n x}{L}\right) = 1$$

$$\therefore \frac{A^2}{2}\left[x + \frac{L}{2\pi n}\sin\left(\frac{2\pi n x}{L}\right)\right]_{-\frac{L}{2}}^{\frac{L}{2}}$$

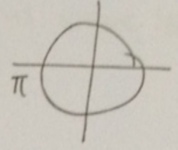
$\Longrightarrow$

$$\frac{A^2}{2}\left(\left[\frac{L}{2} + \frac{L}{2\pi n}\sin\frac{2\pi n L}{2L}\right] - \left[-\frac{L}{2} + \frac{L}{2\pi n}\sin\left(\frac{2\pi n(-L)}{2L}\right)\right]\right) = 1$$

$$= \frac{A^2}{2}\left(\frac{L}{2} + \frac{L}{2\pi n}\sin \pi n + \frac{L}{2} + \frac{L}{2\pi n}\sin \pi n\right) = 1$$

$$= \frac{A^2}{2}\left(L + \underbrace{\frac{L}{\pi n}\sin \pi n}\right) = 1$$

goes to 0 as $\sin n\pi = 0$



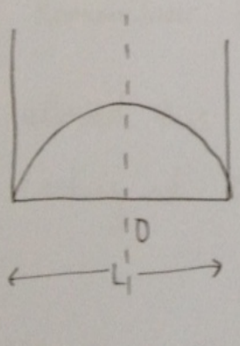
$$\therefore \frac{A^2 L}{2} = 1$$

$$\therefore A = \sqrt{\frac{2}{L}}$$ when you solve for B it turns out $B = \sqrt{\frac{2}{L}}$ as well.

so this gives us $\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi n x}{L}\right)$ when $n = 1, 3, 5 \ldots$ and

$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n x}{L}\right)$ when $n = 2, 4, 6 \ldots$ etc.

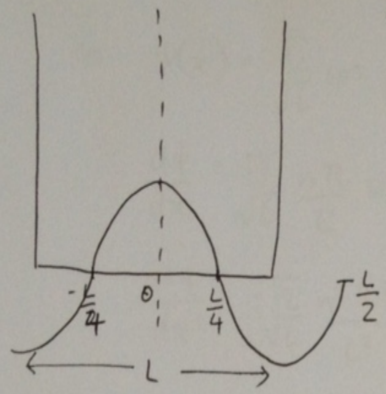
The reason that the system is confined to these specific wave functions arises from the fact that ~~they are~~ the particle is confined to the box. That is, a node must exist at the boundaries of the box: ~~For ex~~ Plotting our wave functions for $n = 1$, we see



$\psi(x) = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L}$ This is a cosine wave with $\lambda = 2L$. This is an allowed wavelength, as nodes exist at the ~~boundary conditions~~ walls of the box.

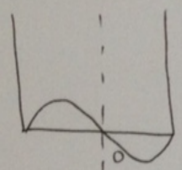
⟹

Let's now consider a cosine wave of ~~$\lambda = 2L$~~ $\lambda = L$

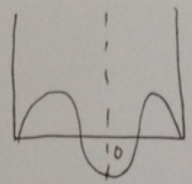


This would imply that the particle may exist <u>outside</u> the box, or that the wave is incomplete at the boundaries. This cannot be true, so this wave length is not allowed.

Just to demonstrate the point further, we will plot $n = 2$ and $n = 3$ to see what "allowed" wavelengths look like.



$\lambda = L$ ~~$\psi = B\sin$~~ $\psi = \sqrt{\frac{2}{L}} \sin \frac{2\pi}{L} x$



$\lambda = \frac{3}{2}$ $\psi = \frac{\sqrt{2}}{\sqrt{L}} \cos \frac{3\pi}{L} x$

Remember that $k = \frac{2\pi}{\lambda}$, so if only certain wavelengths are allowed, this means only certain k values are allowed. Looking at $E = \frac{\hbar^2 k^2}{2m}$, we see that it follows that only certain energy levels are also allowed.

We can use the energy operator to demonstrate this as well.

$\Longrightarrow$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\psi(x)$$

so $\psi(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{n\pi x}{L}\right)$

$$\frac{d\psi}{dx} = -\sqrt{\frac{2}{L}}\frac{n\pi}{L}\sin\left(\frac{n\pi x}{L}\right)$$

$$\frac{d^2\psi}{dx^2} = -\sqrt{\frac{2}{L}}\frac{n^2\pi^2}{L^2}\cos\left(\frac{n\pi x}{L}\right)$$

$$= -\frac{n^2\pi^2}{L^2}\psi(x)$$

$\therefore$ $E = \frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}$ $\Rightarrow$ all terms ~~quan~~ constent except integer n. Energy is quantized.

I want to go on further here to the expanding box to demonstrate the probability densities, but I am still figuring out how to do it.

Feedback is appreciated, plus it would be super helpful if you could grade this just so I know where I'm at.

Thanks Mike. And thanks for teaching us all throughout the semester. You were great.